The Dependence of Laminar Entrance Loss Coefficients on Contraction Ratio for Newtonian Fluids

S. E. KAYE and S. L. ROSEN

Department of Chemical Engineering
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

When a fluid passes in laminar flow through a sharp-edged contraction from a large diameter tube to one of smaller diameter, Sylvester and Rosen (*1, 2*) have shown that the excess entrance pressure loss is given by an equation proposed by Holmes (*3*):

$$\frac{\Delta P_{ent}}{\rho V^2/2g_c} = K + K'/N_{Re} \qquad (1)$$

It was pointed out that the Hagenbach (K) and Couette (K') coefficients must be functions of both β ($= D^2/D_0^2$), the area contraction ratio, and n the fluid's power-law index for *inelastic* fluids. For elastic fluids another dimensionless group, characterizing the degree of elasticity, is presumably required.

Table 2 of reference *1* compared the single experimentally determined values of K and K' for Newtonian fluids ($\beta = 0.0156$, $n = 1$) with previous experimental and theoretical results for Newtonian fluids. On the basis of this comparison, it was stated that both coefficients showed an initial increase with β, which seemed a bit strange because physical reasoning dictates that both coefficients must vanish as β approaches 1. Furthermore, they should reach an asymptotic value as β goes to zero. The last sentence of reference *1* states: "The exact nature of the variation of the coefficients with β requires further study."

In a recent letter, Han (*4*) presents K' versus β data for a polyethylene melt which clearly show a monotonic decrease of K' with β and appears to be heading for the point (0, 1). The effects of fluid elasticity (*2*) on his results cannot be assessed, however, and they are only applicable to the particular material investigated. In fact, it is not clear how his entrance losses are defined for a viscoelastic system in which the radial normal stress varies with radial location.

We have recently obtained experimental results for Newtonian fluids which confirm the monotonic decrease of *both* K and K' with β as well as the asymptote as β goes to zero. The apparatus used is similar to that described previously (*1*), except that the entrance (upstream) tube was lengthened to 42 in. to ensure a 99% fully developed profile at least two diameters in advance of the contraction (*5*). The test (downstream) section had $D = 0.409$ in., and the entrance (upstream) sections ranged from $D_0 = 1/2$ to $D_0 = 2\text{-}1/2$ in., giving β values of 0.636, 0.411, 0.213, 0.109, 0.062, 0.041 and 0.026. The test fluids were aqueous glycerine solutions. Entrance losses were obtained from the equilibrium pressure gradient in the test (downstream) section $(dP/dZ)_T$ and a pressure measurement P_{ES} taken approximately two diameters prior to the contraction

$$\Delta P_{\text{ent}} = P_{ES} - \left| \left(\frac{dP}{dZ}\right)_E \Delta l \right| - \left| \left(\frac{dP}{dZ}\right)_T L_T \right| \qquad (2)$$

The equilibrium gradient in the entrance (upstream) section $(dP/dZ)_E$ was calculated from the equilibrium test-section (downstream) gradient using

$$\left(\frac{dP}{dZ}\right)_E = \beta^2 \left(\frac{dP}{dZ}\right)_T \qquad (3)$$

The correction for friction losses in the entrance tube [the second term on the right of Equation (2)] was significant for the three largest β's.

Figure 1 shows representative results at two values of β, providing excellent confirmation of Equation (1) over the entire laminar range. These curves consist of overlapping data for three different glycerine concentrations, and demonstrate conclusively that K and K' are functions only of the entrance geometry for Newtonian fluids, confirming previous theoretical and experimental work (*1*). From these data, K and K' were obtained using a program which minimized the sum of the squares of the *relative* errors. They are shown as functions of β in Figure 2. Kinetic energy considerations alone predict $K = 2.0\ (1 - \beta^2)$. The least squares curve of this form through the K data is

$$K = 2.32 \pm 0.05\ (1 - \beta^2) \qquad (4)$$

For K', the curve

$$K' = 159 \pm 30\ (1 - \beta^2) \qquad (5)$$

appears to fit the data, although with no theoretical justification. It should be pointed out that the limiting ($\beta = 0$) value of $K = 2.32$ is in perfect agreement with the boundary-layer analysis of Collins and Schowalter (*6*). These results also agree with that of Sylvester and Rosen (*1*), $K = 2.4 \pm 0.1$ ($\beta = .0156$) within experimental precision.

A direct comparison of these Newtonian K' results with those of Han (*2*) is impossible, because his fluid had an $n = 0.28$ (which should lower K') and was highly elastic (which should raise it). His results do not show the expected asymptote as β approaches zero, however. Reanalyzing Sylvester's data (*1*) using the statistical techniques

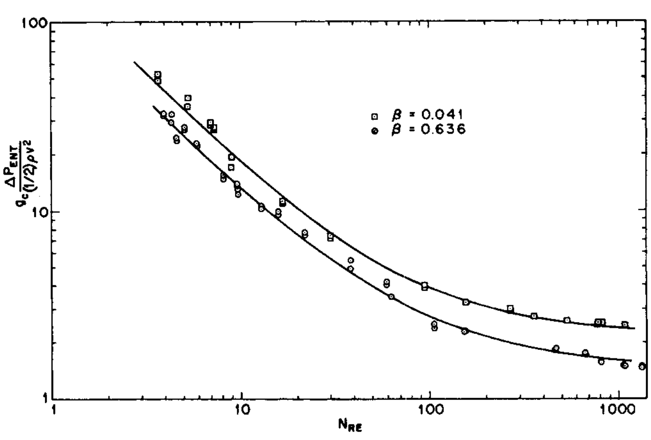

Fig. 1. Dimensionless entrance pressure loss versus Reynolds number for glycerine solutions.

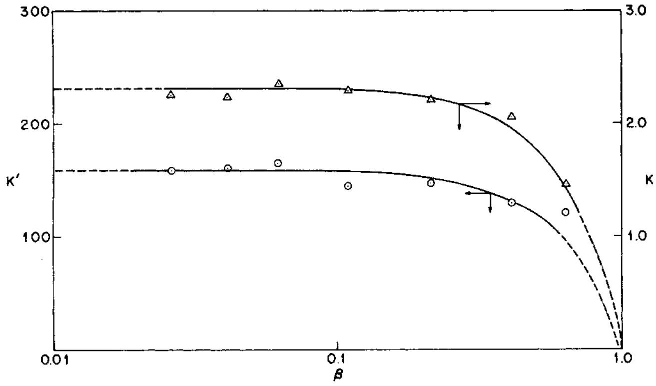

Fig. 2. Couette (*K*) and Hagenbach (*K′*) entrance loss coefficients for square-edged contractions versus area contraction ratio. Curves plotted are Equations (4) and (5).

mentioned above gives a value of K' ($\beta = 0.0156$) $= 241 \pm 50$, which appears to be somewhat higher than the present results. This may be due to the fact that Sylvester's entrance section was not long enough to permit fully developed flow before reaching the contraction, and K' may be particularly sensitive to the nature of the profile in the entrance region.

CONCLUSIONS

The Hagenbach and Couette coefficients for the fully developed laminar flow of a Newtonian fluid through a sharp-edged contraction are given by Equations (4) and (5), respectively. From a practical standpoint, an upstream tube of greater than twice the diameter of the downstream tube may be considered an "infinite" reservoir, $K = 2.32$, $K' = 159$, with less than 4% error.

NOTATION

β = area contraction ratio, $(D/D_0)^2$
D = diameter of test (downstream) tube
D_0 = diameter of entrance (upstream) tube
N_{Re} = Reynolds number in test section
V = volume average velocity in test section
ρ = fluid density
ΔP_{ent} = entrance pressure loss
P_{ES} = pressure (gauge) at location approximately $2\,D_0$ before contraction
Δl = distance from point of measurement of P_{ES} to contraction
L_T = length of test section
$\left(\frac{dP}{dZ}\right)_E$ = equilibrium pressure gradient in entrance section
$\left(\frac{dP}{dZ}\right)_T$ = equilibrium pressure gradient in test section

LITERATURE CITED

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